Theorem: Suppose f is continuous on an interval I that contains one local extremum at c

- If a local min occurs at c, then $f(c)$ is the absolute minimum
- If a local max occurs at c, then $f(c)$ — " — " — maximum.

Eg $f(x) = x^x$, $I = (0, \infty)$

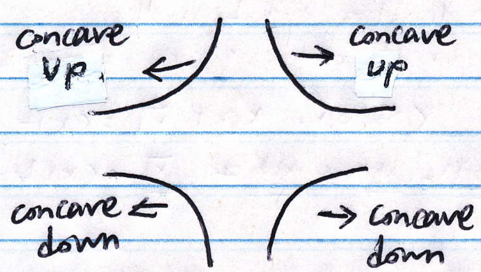
$f(x) = e^{x \ln x}$, $f'(x) = e^{x\ln x}(x \cdot \frac{1}{x} + \ln x)$

$= e^{x \ln x}(1 + \ln x)$

$f'(x) = 0 \Rightarrow (1 + \ln x) = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1} = \frac{1}{e}$; $f'(x)$ exists on $(0, \infty)$

$(0, \infty) = (0, \frac{1}{e}] \cup [\frac{1}{e}, \infty)$, $\underbrace{(0, \frac{1}{e})}_{f'<0}$, $\underbrace{(\frac{1}{e}, \infty)}_{f'>0}$ Hence a local minimum occurs at $x = \frac{1}{e}$

This is the only local minimum, hence it is an absolute minimum with values $f(\frac{1}{e}) \approx 0.69$



§ Concavity and Inflection points

Defn: Let f be differentiable on an open interval I. If f' is increasing on I, then f is concave up on I. If f' is decreasing on I, then f is concave down on I.